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Tilt Determination of Non-Vertical Photography Using a Handheld Programmable Calculator

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An easy to use, inexpensive, and rapid system for determination of tilt in an aerial Motograph has been developed at the Defense Mapping Agency Aerospace Center. Image measurements are made with a calibrated template and tube magnifier. Geodetic control points are then scaled from a large scale map source. Tilt deterwination is then accomplished using Church's resection on a programable calculator, and allows for computation of surface feature lenghts, widths, and helghts from a single image.

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## TILT DETERMINATION OF

## NON-VERTICAL PHOTOGRAPHY

## USING A HANDHELD PROGRAMMABLE CALCULATOR

## By

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## Abstract

A system has been developed which allows the photo interpreter to determine the tilt of an aerial photograph. Input consists of image measurements made using a calibrated template and tube magnifier, and three geodetic control points scaled from large scale map source. Church's resection is used to compute the camera parameters on the Hewlett-Packard HP 41C programmable calculator. Tilt determination allows computation of lengths, widths, and heights from a single image. These calculations follow naturally from the resection and are included in the system.

The system incorporates several desirable features. It is easy to learn and to use, and the resection converges within five minutes. The basic equipment is inexpensive and the calculator may be used for other applications.

## 1. INTRODUCTION:

A photo interpreter or photogrammetrist occasionally is presented nonvertical or oblique photography: as his primary source for photogrammetric measurement. Poor flying conditions or inexpert photographic technique frequently result in undesired nonverticality. Oblique photography may be chosen because a larger area of coverage can be achieved, and a look at the side of a feature to be analyzed permits a more reliable interpretation.

Whatever the source of tilt, the relatively simple measurements of vertical photography become extremely complicated and, without the assistance of a computer, prohibitively time-consuming.

A variety of disciplines rely upon on-the-site field measurements. For many, the level of sophistication does not warrant full-time use of a nonportable computing system. Recent developments in handheld programmable calculators make it possible to quickly resolve the tedious computations needed to determine the tilt of an aerial photograph. These instruments are precise and easy to use. Simple operating instructions and programmed prompts lead the operator through the procedure step-by-step with virtually no opportunity for blundèrs.

A computer program (Appendix B) was developed at the Defense Mapping Agency Aerospace Center (DMAAC) for the Hewlett Packard 41C calculator which determines the tilt of an aerial photograph using Church's Resection (Church, 1945). Horizontal and vertical ground measurements may then be computed for features on that photograph.

## 2. SOFTWARE:

Church's Resection was chosen because the math model, computation algorithm, and pre-tested data were readily available. Professor Earl Church developed the procedure and published it in 1945 in a Syracuse University technical paper. Virtually every theoretical photogrammetry student has cranked out the solution on a desk calculator or has programmed it in FORTRAN. The desk calculator procedure reputedly takes about six hours,
assuming no blunders. Present day students, after struggling through the first iteration, opt for the FORTRAN version (possibly an exercise designed to make students more receptive to the collinearity and coplanarity equations).

Professor Church's solution is iterative, and uses matrices and determinates to derive the exposure station coordinates and the swing and tilt of the aerial photograph (see American Society of Photogrammetry Manual of Photogrammetry, 1980). These values are defined in a rather unique coordinate system. He presents the solution in a "cookbook" form, and provides checks throughout to identify blunders.

Heights (Appendix A.II) and lengths (Appendix A.III) are computed by three digital rectification algorithms. One subroutine computes the direction cosines (Appendix A.I) of vectors from the principal point to the imaged nadir and to the base and top or the end points of the feature being measured. Image coordinates are then projected onto a horizontal plane passing through the nadir image. The vector passing from the principal point through the nadir is substituted for focal length in the conventional single image height and length equations (figures 1, 2, and 3).

The software design features repeated use of the direction cosine algorithm to reduce programming steps and conserve storage.

## 3. HARDWARE:

The HP 4IC programmable calculator (Hewlett-Packard, 1979) was chosen because it had memory expandable to the requirements of the solution and because of the authors' familiarity with the programming language. (Choice of this calculator does not constitute endorsement by DMAAC of Hewlett Packard or the HP 41C calculator.) The HP 41C features continuous memory, LED display, and magnetic card reader. With memory modules, the calculator is expandable to more than 300 registers ( 250 with card reader attached). It is battery operated with very low power consumption, and accessories include rechargeable batteries. The system is suitable for office or field applications.

## 4. PROCEDURE:

Computing heights or lengths with this system consists of measuring map coordinates, measuring image :coordinates, and operating the calculator program. The system concept was developed to increase accuracies with a minimum of operator training and a minimum of opportunity for operator error.

Map derived control is scaled from large scale map source such as United States Geological Survey (USGS) 1:24000 scale topographic maps. The user reads the Northing and Easting from the Universal Transverse Mercator (UTM) grid with, for example, the Metric Coordinate Reader (CR-2) (USGS, 1973), manufactured by USGS. The Geological Survey reports that 15 meter accuracies are expected with their 1:24000 maps.

A 1-centimeter grid aligned to the fiducials of the photo is fixed in place. To measure photo coordinates, the user counts the grid lines to determine centimeter values. Using a Bausch \& Lomb calibrated tube magnifier, he reads $x$ and $y$ coordinates to a least count of 0.1 millimeter. (Choice of the Bausch \& Lomb calibrated tube magnifier does not constitute endorsement by DMAAC of Bausch \& Lomb or its products.) Paper stretch of the photo, image quality, and other fačtors will contribute to error; however, 0.2 millimeter accuracies are expected as a worst case.

To operate the calculator program, the user first runs the routines for Church's Resection (Appendix B, Line O1). Algorithms for length or height determination are then applied.

Church's Resection requires as input: 1) UTM Easting, Northing and height of three widely spaced, non-collinear photoidentifiable control points; 2) the $x$ and $y$ image coordinates of the control; 3) focal length of the camera; and 4) approximate UTM coordinates of the exposure station. These values are input into the calculator's storage registers. The user then starts the program to compute corrections to exposure station coordinates. With reasonable approximations (less than 1000 meters away from the true values), the solution converges after three or four iterations. Corrections to the revised coordinates of the exposure station will be less than one meter.

The operator then executes the command to compute tilt - the angle nO p , imaged nadir, principal point, and imaged principal point; swing - the angle in the photo plane measured clockwise about the optical axis from the $+y$-axis to the imaged nadir; and azimuth of the principal plane - the UTM azimuth of the direction of tilt. These values are automatically stored for use in the following algorithms.

A subroutine called Direction Cosines (DC) (Appendix B, Line 413) computes the vector algebra common to the height and length determinations. The user must input into the storage registers the $x, y$ of the feature of interest and focal length of the camera. That is, the base and top of a vertical feature or the end points of a horizontal feature.

The final step is to command the calculator to execute the height (Appendix B, Line 471) or the length (Appendix B, Line 491) subroutine.

The software was designed for practical photo interpretation applications and will give results comparable to those obtained with conventional vertical photography. Obviously, the geometry is strongest with a 152.43 cm focal length camera and control wideiy spaced to form a strong three-sided pyramid.

Distance measurements are considered to be small, such as the length of a bridge, the edge of a building, or a plot boundary. As with conventional vertical photography, no compensation is made for displacement due to elevation differences.

## 5. CONCLUSION:

The use of tilted aerial photography until now has been largely limited to qualitative applications. A photogrammetrist can use slightly tilted photography on a stereo plotter or the photo interpreter can send it to the photo lab for rectification. However, these procedures require elaborate optical equipment, are time consuming, and require highly skilled technicians.

The tools and procedures for the digital rectification described in this paper open new opportunities for the use of tilted photography. The components are inexpensive, reliable, and virtually maintenance free. The system is easily learned and can be used in either the office or field. Furthermore, a photo interpreter with access to a powerful programmable calculator will develop his own algorithms to make his time more productive.

American Society of Photogrammetry (1980), "Manual of Photogrammetry Fourth Edition", American Society of Photogrammetry, Page 62, 105 Virginia Ave., Falls Church; VA 22046.

Church, Earl (1945), "Revised Geometry of Aerial Photography", Syracuse University Bulletin 15, Syracuse, New York.

Hewlett-Packard (1979), "Owner's Handbook and Programming Guide HP-41C", Hewlett-Packard Company, 1000 N.E. Circle Blvd., Corvallis, OR 97330

United States Department of the Interior Geological Survey (i973), "Metric Coordinate Reader CR-1", United States Department of the Interior Geological Survey, 1340 Old Chain Bridge Road, McLean, VA 22101

## APPENDIX A

## Math Model

## I. Direction Cosines:

Image coordinates of nadir $\mathbf{x}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}}$ and $\mathrm{z}_{\mathrm{n}}$
$x_{n}=\overline{p n} \cos (90-s)$
$y_{n}=\overline{p n} \sin (90-s)$
$z_{n}=-f$
where $\overline{\mathrm{pn}}=\mathrm{f} \tan \mathrm{t}$, $s$ is swing,
$f$ is focal length, $t$ is tilt, and
$\overline{\mathrm{pn}}$ is the vector from the imaged principal point to the imaged nadir.

Computation of direction cosines $1, m$, and $n$.
$l_{i}=x_{i} / d_{i}, \quad m_{i}=y_{i} / d_{i}, \quad n_{i}=z_{i} / d_{i}$
where $d_{i}=x_{i}{ }^{2}+y_{i}{ }^{2}+z_{i}{ }^{2}$
Angles between vectors are computed using the cosine law:
$<v_{1} o v_{2}=\cos ^{-1}\left(1_{1} l_{2}+m_{1} m_{2}+n_{1} n_{2}\right)$
where $\left\langle\mathrm{v}_{1} \mathrm{Ov}\right.$ is the angle at the principal point o between two vectors.

The value $\overline{\text { on }}$ is used in computing both heights and distances.

$$
\begin{equation*}
\overline{\text { on }}=f / \cos t \tag{7}
\end{equation*}
$$

Refer to Figures 1, 2, and 3 for graphic representation of variables.

## 11. Height Determination:

$b^{\prime}$ and $\mathbf{t}^{\prime}$ are points where vectors from the principal point to the base and top of the object being measured would intersect a horizontal plane passing through the imaged nadir.

$$
\begin{align*}
& \overline{\bar{n} b^{\prime}}=\overline{o n} \tan <n o b  \tag{8}\\
& \overline{n t^{\prime}}=\overline{o n} \tan <n o t
\end{align*}
$$

(9)
$d_{i}^{\prime}$ is the rectified image displacement.
$d_{i}^{\prime}=\overline{n t^{\prime}}-\overline{n b^{\prime}}$
$r$ is the rectified radial distance from the imaged nadir to $t \quad$ ectified image of the base of the object.

$$
\begin{equation*}
\mathbf{r}^{\prime}=\overline{n b^{\prime}} \tag{11}
\end{equation*}
$$

H is the height of the exposure station above the object.
$H=Z_{o}-H_{F}$
where $Z_{0}$ is the height of the exposure station above the datum (computed in Church's resection), and
$H_{F}$ is the elevation of the object above the datum.

Substituting the values into the conventional height determination equation

$$
\begin{equation*}
\text { height }=d_{i}^{\prime} H / r^{\prime} \tag{13}
\end{equation*}
$$

## III. Length determination:

The equation for the length of an object on a conventional vertical aerial photograph is

$$
\begin{equation*}
L=1\left(Z_{0}-H\right) / f \tag{14}
\end{equation*}
$$

where $L$ is the length of the object 1 is the imaged length of the object and the other parameters are as defined before.

Using the horizontal plane passing through the imaged nadir we can substitute the rectified image values:

$$
\begin{equation*}
L=I^{\prime}\left(Z_{0}-H\right) / \overline{o n} \tag{15}
\end{equation*}
$$

where $l^{\prime}$ is the rectified image length.
$I^{\prime}$ is the vector subtraction of the vectors $\overline{\mathrm{ce}}{ }^{\prime}$ and $\overline{\mathrm{oe}}{ }_{2}{ }^{\prime}$, where $\mathrm{e}_{1}{ }^{\prime}$ and $e_{2}$ are the rectified image points of the ends of the object being measured.

$$
\begin{align*}
I^{\prime} & ={\overline{o e_{2}}}_{2}-{\overline{o e_{1}}}_{1}^{\prime}  \tag{16}\\
& =\overline{\mathrm{oe}}_{1}{ }^{2}+\overline{\mathrm{oe}}_{2}{ }^{2}-2 \overline{\mathrm{oe}}_{1} \overline{\mathrm{oe}}_{2} \cos <\mathrm{e}_{2} \mathrm{oe}_{1} \tag{17}
\end{align*}
$$

$$
\begin{align*}
& \overline{\mathrm{oe}}_{1}^{\prime}=\overline{o n} / \sin <\text { noe }_{1} \quad \text { and }  \tag{18}\\
& \overline{\mathrm{oe}}_{2}^{\prime}=\overline{\mathrm{on}} / \sin <\text { noe }_{2} \tag{19}
\end{align*}
$$

## APPENDIX B

Program Listing


40 STO 56

41 XEQ 14
: 27.02601 STO 56 -1 STO 57
XEQ 04
26.03801

STO 57
. 00801
STO 56
XEQ 13
27.03501

STO 57
XEQ 06
XEQ 12
27.02901

STO 57
54.05001

STO 58
42.04401

STO 56
52
STO 59
55
STO 60 43 STO 61 XEQ 07 BEEP
View 44 PSE
View 43
PSE
View 42
STOP
XEQ 11
GTO 58 SF 00 GTO 58 STOP

81 LBL 15
CLX
LBL 50
RCL IND 56
X / 2
$+$
ISG 56
GTO 50
SQRT
90 1/X
1ES
X < > Y
*
STO IND 57
LAST $X$
3
ST- 56
RDN
LBL 51
STO IND 56
ISG 56
GTO 51
. 003
ST +56
ISG 57
GTO 15
RTN
LBL 02
RCL 39
110 RCL 51
100000
*TO 51
RCL 40
RCl 52
100,000
*
120 STO 52

121 RCL 41
RCL 53
100000
*
STO 53
RTN
LBL 03
130 STO IND 57
.003
ST + 59
LBL 52
RCL 59
3
$+$
9
MOD
STO 58
140 RCL IND 59
RCL IND 58
*
STO IND 57
ISG 59
GTO 52
ISG 57
GTO 03
CLX
STO 42
150 STO 44
STO 46
RTN
LBL 04
.003
STO + 56
1
STO +57
42.04801

STO 58
160 LBL 53

161 RCL IND 57 RCL IND 58 *

STO IND 56
RCL 57
$-3$
,
9
MOD
170 STO 57
ISG 58
RCL IND 57
RCL IND 58
*
STO IND 56
ISG 58
ISG 56
GTO 53
RCL 56
18035
$\mathrm{X}>\mathrm{Y}$ ?
GTO 04
RTN
LBL 14
RCL 48
ENTER /
ENTER /
RCL 39
RCL 49
190 XEQ 54
RCL 40
RCL 50
XEQ 54
RCL 41
RCL 48
LBL 54
ST-IND 56
RDN
*
200 ST + IND 56

201 ISG 56 $X<>Y$ LAST X R
*
STO IND 56
RDN
ST- IND 56
LAST X
210 ENTER /
ISG 56
RTN
RTN
LBL 06
RCL 01
RCL 03
RCL 00
RCL 04
XEQ 05
220 RCL 02
RCL 03
RCL 00
RCL 05
XEQ 05
RCL 02
RCL 04
RCL 01
RCL 05
XEQ 05
230 RCL 01
RCL 06
RCL 00
RCL 07
XEQ 05
RCL 02
RCL 06
RCL 08
RCL 00
XEQ 05
240 RCL 07

241 RCL 02
RCL 01
RCL 08
XEQ 05
RCL 06
RCL 04
RCL 03
RCL 07
XEQ 05
250 RCL 05
RCL 06
RCL 03
RCL 08
XEQ 05
RCL 05
RCL 07
RCL 04
RCL 08
LBL 05
260
STO IND 57 RDN
*
ST- IND 57
ISG 57
RTN
RTN
LBL 07
RCL IND 59
270 CHS
STO IND 59
LBL 56
0
STO IND 56
LBL 57
DSE 58
RCL IND 57
RCL IND 58 *
280 ST + IND 56

281 ISG 57
GTO 57
RCL IND 60
ST/IND 56
:. 003
ST +57
8.003

FS? 00
ST- 57
290
$\mathbf{S T}+58$
6
FS? 00
ST+ 58
ISG 56
GTO 56
RCL IND 61
CHS
STO IND 61
300 RCL IND 59
CHS
STO IND 59
FC? 00
RTN
1
ST+ 58
ST+ 59
9.01703

STO 57
310.003
$\mathrm{ST}+56$
ISG 61
GTO 07
RTN
LBL 08
RCL IND 56
RCL IND 57
STO IND 59
320

321 ST+ 56
ST+ 57 ISG 59 GTO 08 .003
ST +59
3
ST- 57
ISG 58
330 GTO 08
RTN
LBL, 09
RCL 09
RCL 08
*
RCL 12
RCL 05
*
-
340 RCL 15
RCL 02
*
$+$
STO 54
RTN
LBL 10
-1.00901
STO 57
18.02601

350 STO 56
XEQ 13
9.01703

STO 57
27.01703

STO 58
. 00201
STO 56
21
STO 59
$360 \quad 54$

361 STO 60
1.00703

STO 61
XEQ 07
GTO 16
$\frac{\mathrm{LBL}}{\mathrm{RCI}}{ }_{44}$
$\mathrm{ST}+36$
RCL 43
370 ST+ 37
RCL 42
ST+ 38
RTN
LBL 12
RCL 27
RCL 08
*
RCL 28
RCL 07
380 *
RCL 29
RCL 06
*
$+$
STO 55
RTN
LBL 13
ISG 57
390 RCL IND 57
STO IND 56
ISG 56
GTO 13
RTN
$\frac{\text { LBL }}{\text { RCL }}{ }^{16}$
ACOS
STO 39 360
400 RCL 08

401
RCL 07 1
ATAN
$\mathrm{X}<\mathrm{O}$ ?
: ${ }^{\text {STO }} 40$
RCL 00
RCL 03
1
410 ATAN
STO 41
STOP
LBL $^{\text {T }}$ DC
RCL 14
RCL 40
TAN
*
STO 15
RCL 40
420
90
$+$
STO 16
cos
*
STO 00
RCL 15
RCL 16
SIN
*
431 STO 01
RCL 14
STO 02
STO 05
STO 08
0.00201

STO 56
48.05001

STO 57
XEQ 15
441 . 00201

442 STO 56 3.00901

STO 57
12.01301

STO 55
LBL 18
CLX
STO IND 55
LBL 17
451 RCL IND 56
RCL IND 57
$\stackrel{*}{*}$
ST+ IND 55
ISG 57
ISG 56
GTO 17
ACOS
STO IND 55
. 00201
461 STO 56
ISG 55
GTO 18
RCL 14
CHS
RCL 39
COS
1
STO 17
STOP

TAN
*
STO 13
RCL 17
RCL 12
TAN
*TO 12
RCL 13
482

| 483 | STO 11 | 520 | * |
| :---: | :---: | :---: | :---: |
|  | RCL 38 |  | 2 |
|  | RCL 28 |  | * |
|  | - |  | CHS |
|  | * |  | : RCL 12 |
|  | RCL 12 |  | X / 2 |
|  | 1 |  |  |
| 490 | STOP |  | RCL 13 |
|  | $\mathrm{LBL}^{\text { }}$ L |  | X / 2 |
|  | RCL 17 |  | + |
|  | RCL 12 | 530 | SQRT |
|  | SIN |  | STO 19 |
|  | 1 |  | RCL 38 |
|  | STO 12 |  | STO 19 |
|  | RCL 17 |  | RCL 27 |
|  | RCL 13 |  | RCL 28 |
|  | SIN |  | - |
| 500 |  |  | * |
|  | STO 13 |  | RCL 17 |
|  | 3.00501 |  | 1 |
|  | STO 56 | 540 | STOP |
|  | 6.00901 |  | END |
|  | STO 57 |  |  |
|  | CLX |  |  |
|  | STO 18 |  |  |
|  | LBL 19 |  |  |
|  | RCL IND 56 |  |  |
| 510 | RCL IND 57 |  |  |
|  | * |  |  |
|  | STO 18 |  |  |
|  | ISG 57 |  |  |
|  | ISG 56 |  |  |
|  | GTO 19 |  |  |
|  | RCL 12 |  |  |
|  | RCL 13 |  |  |
|  | * |  |  |
|  | RCL 18 |  |  |



Figure 1


Photo plane of tilted aerial photograph showing swing, imaged nadir, and imaged vertical feature.
Low oblique photo viewed in the plane nop

Figure 2


The plane passing through the base and top of the feature, the nadir, and the principal point, plane ONTB.

Figure 3


Figure 4


FIGURE 5. The CR-2 Metric Coordinate Reader


FIGURE 6. Bausch \& Lomb Tube Magnifier


FIGURE 7. Measuring Photo Coordinates

Roberta L. Braundel is a Cartographer (Photogrammetry) in the Techniques Office of the Air Target Materials/Nav-Plan Charting Division of the Aerospace Cartography Department at the Defense Mapping Agency Aerospace Center. She received her bachelors degree in Data Information Systems from Tarkio College and is currently working on her masters degree in Computer Science at the University of Missouri/Rolla. She is currently assisting in the development of various automated techniques associated with map and chart production as well as in the development of automated methods of chart production scheduling and control. Mrs. Braundel is a member of the American Congress on Surveying and Mapping and serves as Secretary of the St. Louis Section - ACSM.

Philip K. Alderman received his bachelors degree in mathematics in 1967 from Southern State College in Arkansas. He received his masters degree in Civil Engineering (Photogrammetry) in 1971 from the University of Illinois. His mapping career includes experience as a cartographer, analog and analytical Stereo Plotters, Aerial Photo-Inspector, instructor at the DMAIAGS Cartographic School and project engineer in the DMAIAGS Bolivia Project. He is currently assigned to the Techniques Office of the Air Target Materials/ Nav-Plan Charting Division of the Aerospace Cartography Department at the Defense Mapping Agency Aerospace Center. Mr. Alderman is a member of the American Society of Photogrammetry and the American Congress on Surveying and Mapping.

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